University of Saskatchewan
Department of Mathematics & Statistics
Mathematics 124.3 (02, 04, 06, 08)

Time: 3 hours — Final Examination — April 23, 2004

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Name: _____ Student #: _____ Math 124 section #____

No books, notes or calculators are allowed.

PART A

For the questions in Part A, simply write down your answer in the space provided. Use one of the examination booklets to write out your complete solution. Number your solutions carefully, and please write neatly.
Each question in Part A is worth 3 marks.

1. Find $\int_{-2}^{2} \sqrt{4-x^2}\,dx$ by interpreting it in terms of an area.

   Answer: _____

2. If $F(x) = \int_{0}^{x^2+\pi} t\cos t\,dt$, what is $F'(x)$?

   Answer: _____

3. Evaluate the Riemann Sum approximation to $\int_{1}^{9} \frac{1}{x+1}\,dx$ using 4 subdivisions and the left hand points of each subdivision as the sample points. Give your answer as a single rational number $\frac{p}{q}$.

   Answer: _____

4. Use Simpsons' Rule with 4 subdivisions to approximate $\int_{1}^{9} \frac{1}{x+1}\,dx$. Give your answer as a single rational number.

   Answer: _____

5. Find the area of the region enclosed by the curves $y = x^2$ and $y^2 = x$.

   Answer: _____

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6. Evaluate the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = 1$ about the y-axis.

   Answer: _____

7. Evaluate the volume of the solid obtained when the region in question 6 is rotated about the x-axis.

   Answer: _____

In questions 8 to 14 evaluate the integrals.

8. $\displaystyle\int \sin^2 x \cos^3 x \, dx =$

   Answer: _____

9. $\displaystyle\int x^3 \sqrt{1 + x^2} \, dx =$

   Answer: _____

10. $\displaystyle\int \frac{2}{(x-1)(x+1)} \, dx =$

    Answer: _____

11. $\displaystyle\int \frac{x}{\sqrt{3 - x^4}} \, dx =$

    Answer: _____

12. $\displaystyle\int \cos^{-1} x \, dx =$

    Answer: _____

13. $\displaystyle\int \frac{1 + \ln x}{x \ln x} \, dx =$

    Answer: _____

14. $\displaystyle\int_1^2 x^3 \ln x \, dx =$

    Answer: _____

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15. What is the value of the improper integral $\displaystyle\int_0^1 \frac{dx}{(1-x)^{\frac{2}{3}}}$.

Answer: _____

16. Find the solution of the differential equation

$$\frac{du}{dt} = \frac{2t + \sec^2 t}{2u}, \quad u(0) = -5 \ .$$

Answer: _____

In questions 17 and 18 consider the curve given parametrically by $x = \cos^3\theta$, $y = \sin^3\theta$, $0 \le \theta \le \frac{\pi}{2}$.

17. What is the length of the curve?

Answer: _____

18. What is the area of the surface generated by rotating the curve about the x-axis?

Answer: _____

19. An airtight cubic box 2m × 2m × 2m is to be constructed of plate steel. The box is to hold a video camera and is going to be used to photograph the wreck of the Titanic. If the water depth is 2,000 m, calculate the hydrostatic force that will be exerted on one of the vertical sides of the box when it rests on the bottom of the Atlantic Ocean beside the Titanic. (Use water mass density = 1,000 kg/m^3 and use g for gravitational acceleration. Leave your answer in the form of a number times g, like $xxxg$.]

Answer: _____

20. The linear density of a rod 8m long is $\dfrac{12}{\sqrt{x+1}}$ kg/m, where x is measured in meters from one end of the rod. Find the average density of the rod.

Answer: _____

[60] Total marks for `Part A`.

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Part B

For questions in Part B, write out the complete solution in one of the examination booklets. Show all of your work and please write neatly.

Each question in Part B is worth 10 marks.

21. Evalute $\displaystyle\int \frac{dx}{x^4 - 16}$.

22. In 1980 a new golf course was carved out of the prairie landscape. It was estimated that there were only 100 gophers on the course and they were considered to be no problem. By 1990 it was estimated that the gopher population had grown to 900. If the rate of population growth of gophers is proportional to the population size, when will the population be 24,300?

23. A parabolic tank of height 4m can be considered as being generated by rotating the curve $y = \frac{1}{4}x^2$ about the y-axis. The tank is full of water which has mass density of 1000 kg/m^3. How much work is required to pump all of the water out of the tank to a height of 2m above the top of the tank.

24. (a) On a single graph, sketch the polar curves $r = 2\cos 2\theta$ and $r = 1$. Use the convention that r can be positive or negative.

    (b) Find the area inside the curve $r = 2\cos 2\theta$ and outside the curve $r = 1$.

[40] Total marks for `Part B`.

The End